CMSC 206

Dictionaries and Hashing

The Dictionary ADT

- a dictionary (table) is an abstract model of a database or lookup table
- like a priority queue, a dictionary stores key-element pairs
- the main operation supported by a dictionary is searching by key

Examples

- Telephone directory
- Library catalogue
- Books in print: key ISBN
- FAT (File Allocation Table)

The Dictionary ADT

- simple container methods:
  - size()
  - isEmpty()
  - iterator()
- query methods:
  - get(key)
  - getAllElements(key)

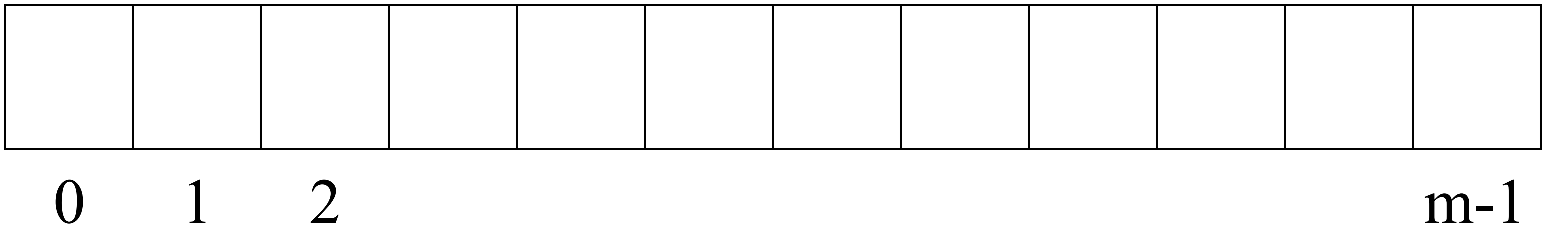
The Dictionary ADT

- update methods:
  - insert(key, element)
  - remove(key)
  - removeAllElements(key)
- special element
  - NO_SUCH_KEY, returned by an unsuccessful search

The Basic Problem

- We have lots of data to store.
- We desire efficient – O(1) – performance for insertion, deletion and searching.
- Too much (wasted) memory is required if we use an array indexed by the data's key.
- The solution is a "hash table".

Hash Table



- Basic Idea
  - The hash table is an array of size 'm'
  - The storage index for an item determined by a *hash function* h(k): $U \rightarrow \{0, 1, \ldots, m-1\}$
- Desired Properties of h(k)
  - easy to compute
  - uniform distribution of keys over $\{0, 1, \ldots, m-1\}$
    - when $h(k_1) = h(k_2)$ for $k_1, k_2 \in U$, we have a *collision*

Division Method

- The hash function:

  h(k) = k mod m where m is the table size.

- m must be chosen to spread keys evenly.
  - Poor choice: m = a power of 10
  - Poor choice: $m = 2^b$, b> 1
- A good choice of m is a prime number.
- Table should be no more than 80% full.
  - Choose m as smallest prime number greater than m_{min}, where
    m_{min} = (expected number of entries)/0.8

Multiplication Method

- The hash function:

  $h(k) = \lfloor m(kA - \lfloor kA \rfloor) \rfloor$

  where A is some real positive constant.
- A very good choice of A is the inverse of the "golden ratio."
- Given two positive numbers x and y, the ratio x/y is the "golden ratio" if $\phi = x/y = (x+y)/x$
- The golden ratio:

  $x^2 - xy - y^2 = 0 \Rightarrow \phi^2 - \phi - 1 = 0$

  $\phi = (1 + sqrt(5))/2 = 1.618033989\ldots$

  $\sim= Fib_i/Fib_{i-1}$

Multiplication Method (cont.)

- Because of the relationship of the golden ratio to Fibonacci numbers, this particular value of A in the multiplication method is called "Fibonacci hashing."
- Some values of

$$h(k) = \lfloor m(k\,\phi^{-1} - \lfloor k\,\phi^{-1} \rfloor)\rfloor$$

= 0 for k = 0

= 0.618m for k = 1 (ϕ^{-1} = 1/ 1.618… = 0.618…)

= 0.236m for k = 2

= 0.854m for k = 3

= 0.472m for k = 4

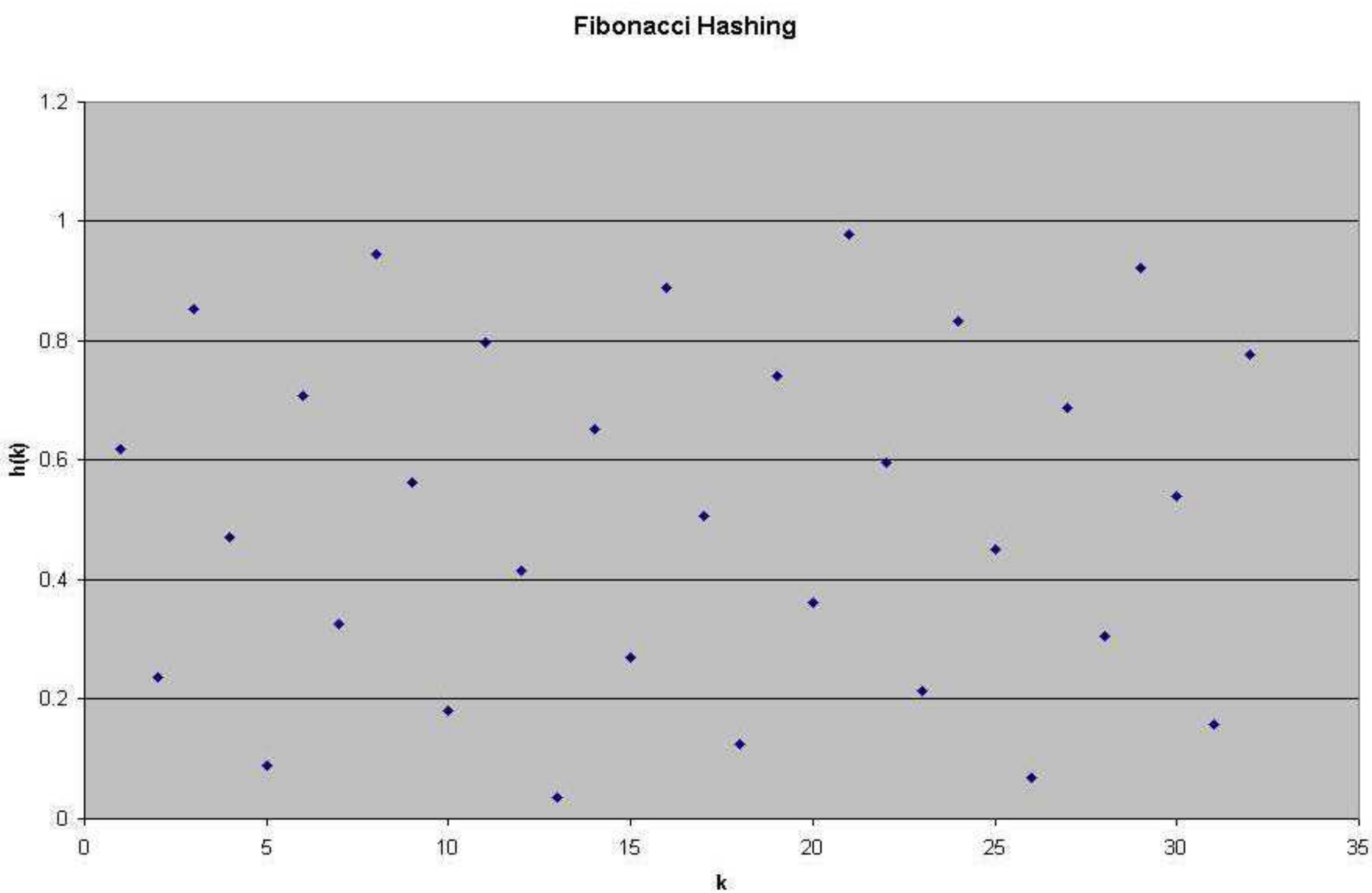
= 0.090m for k = 5

= 0.708m for k = 6

= 0.326m for k = 7

= …

= 0.777m for k = 32


Fibonacci Hashing
h(k)
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
30
35
k

Non-integer Keys

- In order to have a non-integer key, must first convert to a positive integer:

$$h(k) = g(f(k)) \text{ with } f: U \rightarrow \text{integer}$$
$$g: I \rightarrow \{0 .. m-1\}$$

- Suppose the keys are strings.
- How can we convert a string (or characters) into an integer value?

Horner’s Rule

```
static int hash(String key, int tableSize)
{
  int hashVal = 0;

  for (int i = 0; i < key.length(); i++)
     hashVal = 37 * hashVal + key.charAt(i);

  hashVal %= tableSize;
  if(hashVal < 0)
     hashVal += tableSize;

  return hashVal;
}
```

Example:

```
value = (s[i] + 31*value) % 101;
```

- A. **Aho**, J. Hopcroft, J. Ullman, *"Data Structures and Algorithms"*, 1983, Addison-Wesley.

'A' = 65 'h' = 104 'o' = 111

```
value = (65 + 31 * 0) % 101 = 65
value = (104 + 31 * 65) % 101 = 99
value = (111 + 31 * 99) % 101 = 49
```

Example:

```
value = (s[i] + 31*value) % 101;
```

Key	Hash Value
Aho	49
Kruse	95
Standish	60
Horowitz	28
Langsam	21
Sedgewick	24
Knuth	44

resulting table is "sparse"

Example:

```
value = (s[i] + 1024*value) % 128;
```

Key	Hash Value
Aho	111
Kruse	101
Standish	104
Horowitz	122
Langsam	109
Sedgewick	107
Knuth	104

likely to result in "clustering"

Example:

```
value = (s[i] + 3*value) % 7;
```

Key	Hash Value
Aho	0
Kruse	5
Standish	1
Horowitz	5
Langsam	5
Sedgewick	2
Knuth	1

"collisions"

HashTable Class

```
public class SeparateChainingHashTable<AnyType>
{
    public SeparateChainingHashTable( ){/* Later */}
    public SeparateChainingHashTable(int size){/*Later*/}
    public void insert( AnyType x ){ /*Later*/ }
    public void remove( AnyType x ){ /*Later*/}
    public boolean contains( AnyType x ){/*Later */}
    public void makeEmpty( ){ /* Later */ }
    private static final int DEFAULT_TABLE_SIZE = 101;
    private List<AnyType> [ ] theLists;
    private int currentSize;
    private void rehash( ){ /* Later */ }
    private int myhash( AnyType x ){ /* Later */ }
    private static int nextPrime( int n ){ /* Later */ }
    private static boolean isPrime( int n ){ /* Later */ }
}
```

HashTable Ops

- `boolean contains( AnyType x )`
  - Returns true if x is present in the table.
- `void insert (AnyType x)`
  - If x already in table, do nothing.
  - Otherwise, insert it, using the appropriate hash function.
- `void remove (AnyType x)`
  - Remove the instance of x, if x is present.
  - Ptherwise, does nothing
- `void makeEmpty()`

Hash Methods

```
private int myhash( AnyType x )
{
        int hashVal = x.hashCode( );

        hashVal %= theLists.length;
        if( hashVal < 0 )
            hashVal += theLists.length;

        return hashVal;
}
```

Handling Collisions

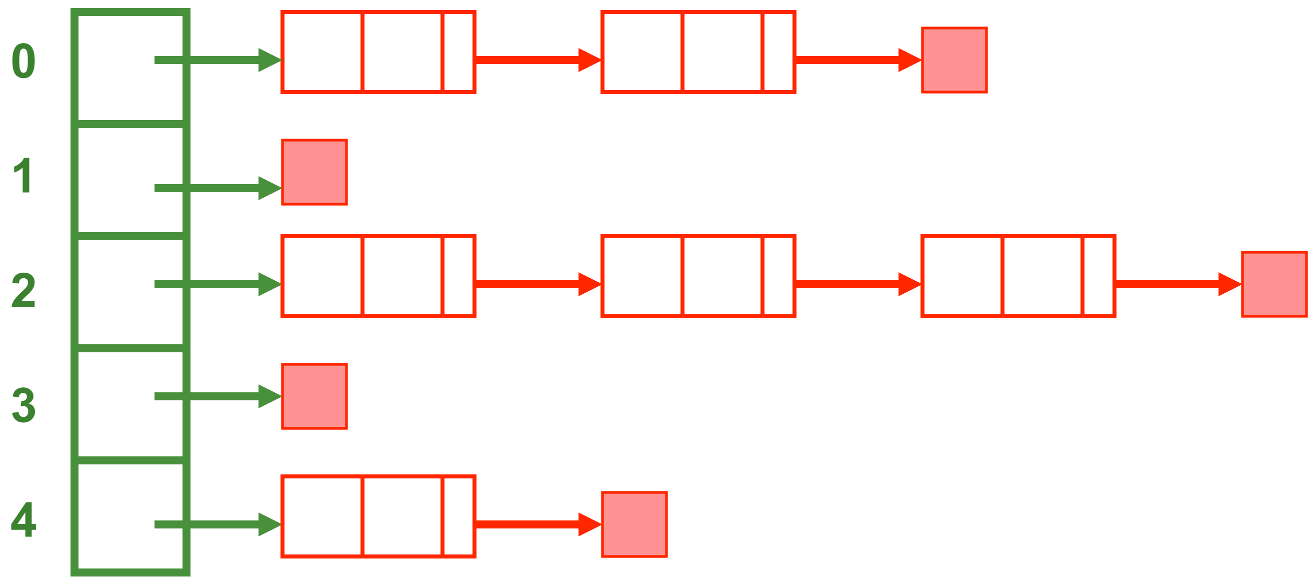
- Collisions are inevitable. How to handle them?
- Separate chaining hash tables
  - Store colliding items in a list.
  - If m is large enough, list lengths are small.
- Insertion of key k
  - hash(k) to find the proper list.
  - If k is in that list, do nothing, else insert k on that list.
- Asymptotic performance
  - If always inserted at head of list, and no duplicates, insert = O(1) for best, worst and average cases

Hash Class for Separate Chaining

- To implement separate chaining, the private data of the hash table is an array of Lists. The hash functions are written using List functions

```
private List<AnyType> [ ] theLists;
```

Chaining



Performance of contains( )

- contains
  - Hash k to find the proper list.
  - Call contains( ) on that list which returns a boolean.
- Performance
  - best:
  - worst:
  - average

Performance of remove( )

- Remove k from table
  - Hash k to find proper list.
  - Remove k from list.
- Performance
  - best
  - worst
  - average

Handling Collisions Revisited

- ***Probing hash tables***
  - All elements stored in the table itself (so table should be large. Rule of thumb: m >= 2N)
  - Upon collision, item is hashed to a new (open) slot.
- Hash function

  h: U x {0,1,2,….} → {0,1,…,m-1}

  h(k, i) = (h’ (k) + f(i)) mod m

  for some h’ : U → { 0, 1,…, m-1}

  and some f(i) such that f(0) = 0
- Each attempt to find an open slot (i.e. calculating h(k, i)) is called a ***probe***

HashEntry Class for Probing Hash Tables

- In this case, the hash table is just an array

```
private static class HashEntry<AnyType>{
   public AnyType  element;  // the element
   public boolean isActive;  // false if deleted
   public HashEntry( AnyType e )
   { this( e, true ); }
   public HashEntry( AnyType e, boolean active )
   { element  = e; isActive = active; }
}
// The array of elements
private HashEntry<AnyType> [ ] array;
// The number of occupied cells
private int currentSize;
```

Linear Probing

- Use a linear function for f(i)

$$f(i) = c * i$$

- Example:

  h' (k) = k mod 10 in a table of size 10 , f(i) = i

So that

  h(k, i) = (k mod 10 + i) mod 10

  Insert the values U={89,18,49,58,69} into the hash table

Linear Probing (cont.)

- Problem: Clustering
  - When the table starts to fill up, performance → O (N)

- Asymptotic Performance
  - Insertion and unsuccessful find, average
    - λ is the "load factor" – what fraction of the table is used
    - Number of probes ≅ $(\frac{1}{2}) (1+1/(1-\lambda)^2)$
    - if λ ≅ 1, the denominator goes to zero and the number of probes goes to infinity

Linear Probing (cont.)

- Remove
  - Can't just use the hash function(s) to find the object and remove it, because objects that were inserted after X were hashed based on X's presence.
  - Can just mark the cell as deleted so it won't be found anymore.
    - Other elements still in right cells
    - Table can fill with lots of deleted junk

Linear Probing Example

- h(k) = *k* mod 13
- Insert keys:
- 18 41 22 44 59 32 31 73

0	1	2	3	4	5	6	7	8	9	10	11	12

		41			18	44	59	32	22	31	72	
0	1	2	3	4	5	6	7	8	9	10	11	12

Quadratic Probing

- Use a quadratic function for f(i)

  $$f(i) = c_2 i^2 + c_1 i + c_0$$

  The simplest quadratic function is $f(i) = i^2$

- Example:

  Let $f(i) = i^2$ and $m = 10$

  Let $h'(k) = k \bmod 10$

  So that

  $$h(k, i) = (k \bmod 10 + i^2) \bmod 10$$

  Insert the value U={89, 18, 49, 58, 69 } into an initially empty hash table

Quadratic Probing (cont.)

- Advantage:
  - Reduced clustering problem
- Disadvantages:
  - Reduced number of sequences
  - No guarantee that empty slot will be found if $\lambda \geq 0.5$, even if m is prime
  - If m is not prime, may not find an empty slot even if $\lambda < 0.5$

Double Hashing

- Let f(i) use another hash function

$$f(i) = i * h_2(k)$$

Then h(k, I) = (h'(k) + i * h_2(k)) mod m

And probes are performed at distances of

h_2(k), 2 * h_2(k), 3 * h_2(k), 4 * h_2(k), etc

- Choosing h_2(k)
  - Don't allow h_2(k) = 0 for any k.
  - A good choice:
    h_2(k) = R - (k mod R) with R a prime smaller than m
- Characteristics
  - No clustering problem
  - Requires a second hash function

Rehashing

- If the table gets too full, the running time of the basic operations starts to degrade.
- For hash tables with separate chaining, "too full" means more than one element per list (on average)
- For probing hash tables, "too full" is determined as an arbitrary value of the load factor.
- To rehash, make a copy of the hash table, double the table size, and insert all elements (from the copy) of the old table into the new table
- Rehashing is expensive, but occurs very infrequently.