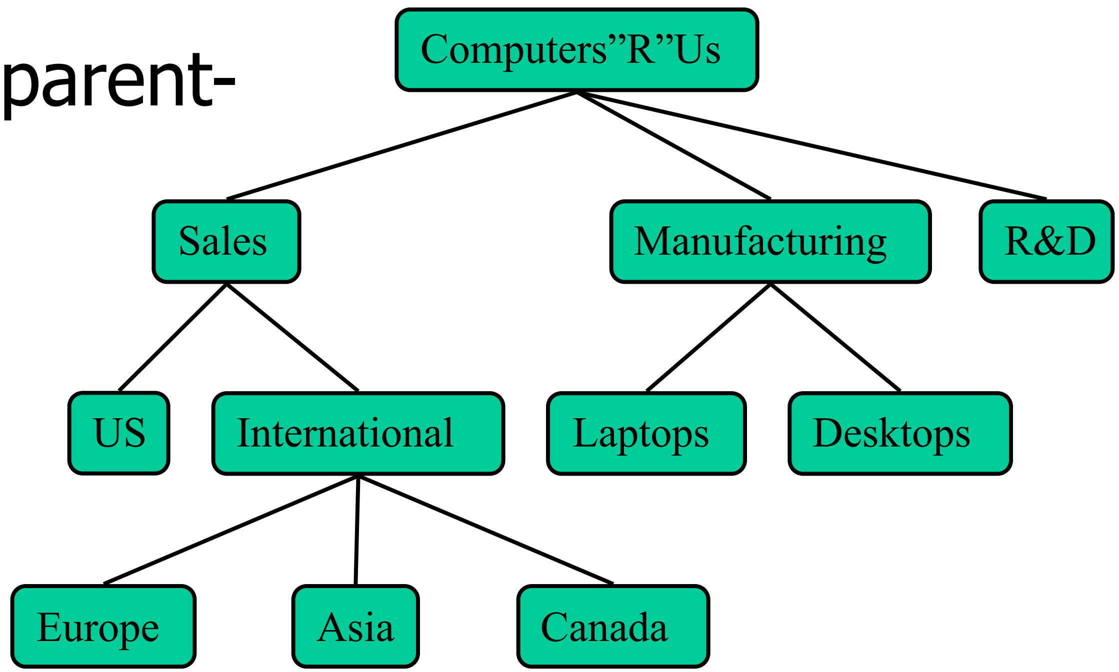
CS206

Trees

Tree

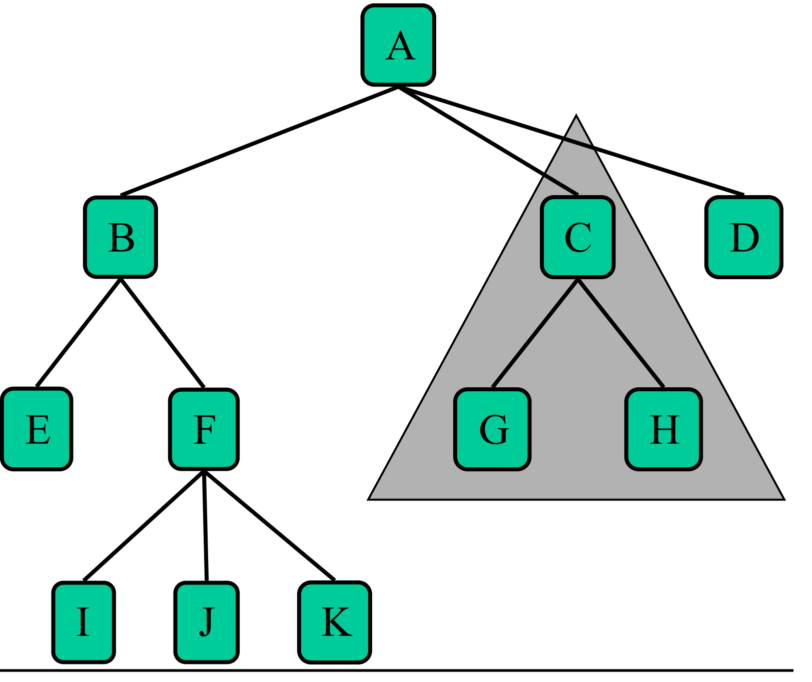
- A tree is an abstract model of a hierarchical structure
- Nodes have a parent-child relation



Terminology

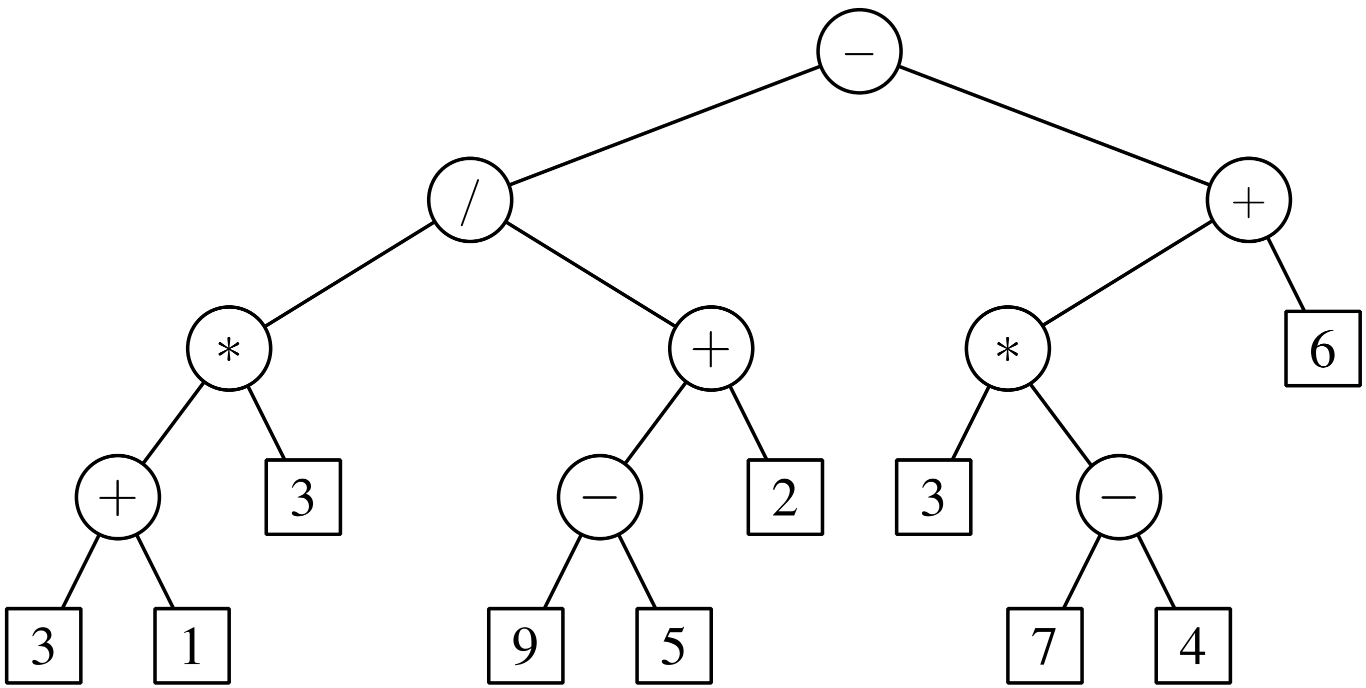
- root: no parent – A
- external node/leaf: no children – E, I, J, K, G, H, D
- internal node: - node with at least one child - A, B, C, F
- ancestor/descendent
- depth - # of ancestors
- Height - max depth

- Subtree: tree consisting of a node and its descendants



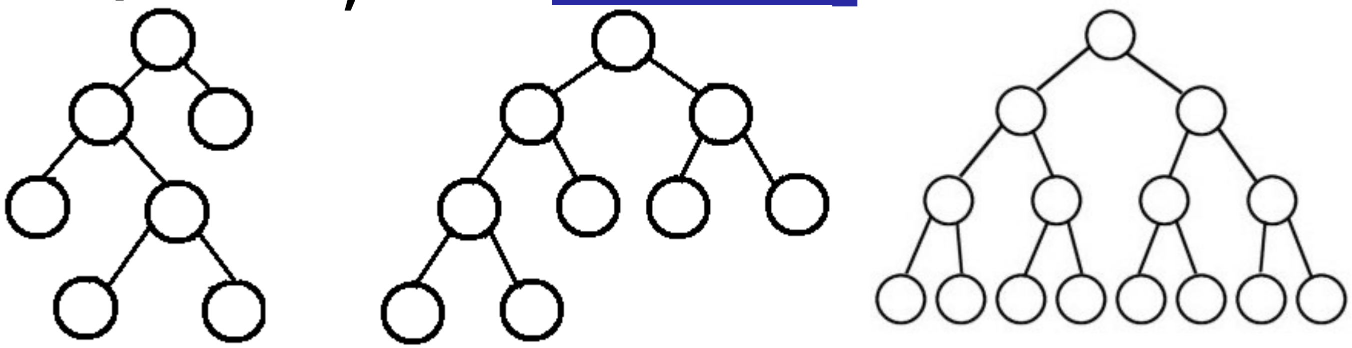
Binary Tree

- An ordered tree with every node having at most two children – left and right



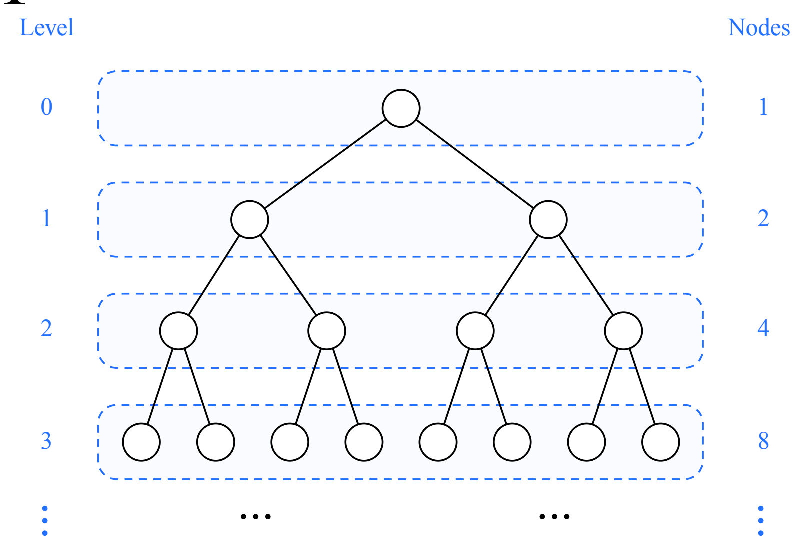
Type of Binary Trees

- A binary tree is **proper** (or full) if each node has zero or two children
- A binary tree is **complete** if every level (except possibly the last) is filled
- If a complete binary tree is filled at every level, it is **perfect**



Binary Tree Properties

- Let n denote the number of nodes and h the height of a binary tree
  - $h + 1 \leq n \leq 2^{h+1} - 1$
  - $\log(n + 1) - 1 \leq h \leq n - 1$
- Height of a binary tree is usually $O(logn)$ of the max number of nodes — true worst case O(n)



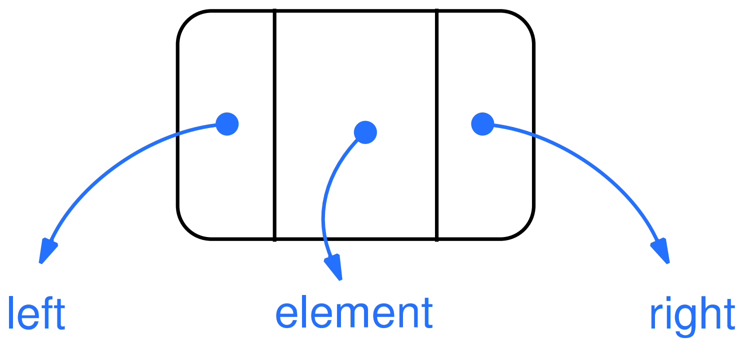
Interface

```java
public interface BinaryTreeInterface<B>
{
    int size();
    int height();
    boolean isEmpty();
    boolean contains(B element);
    void insert(B element);
    B remove(B element);
}
```

Implementation

```
private class Node {
    E payload;
    Node right;
    Node left;

    public Node(E e) {
        payload=e;
        right=null;
        left=null;
    }
    public String toString() {
        return payload.toString();
    }
}
```



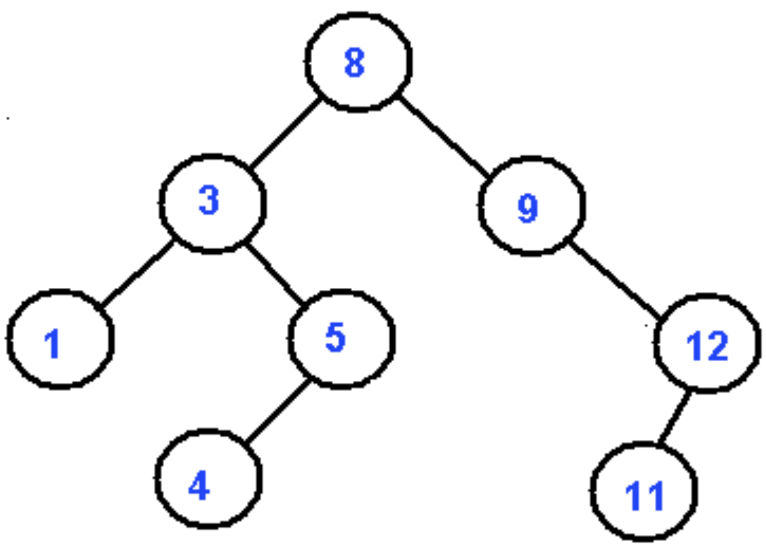
Class

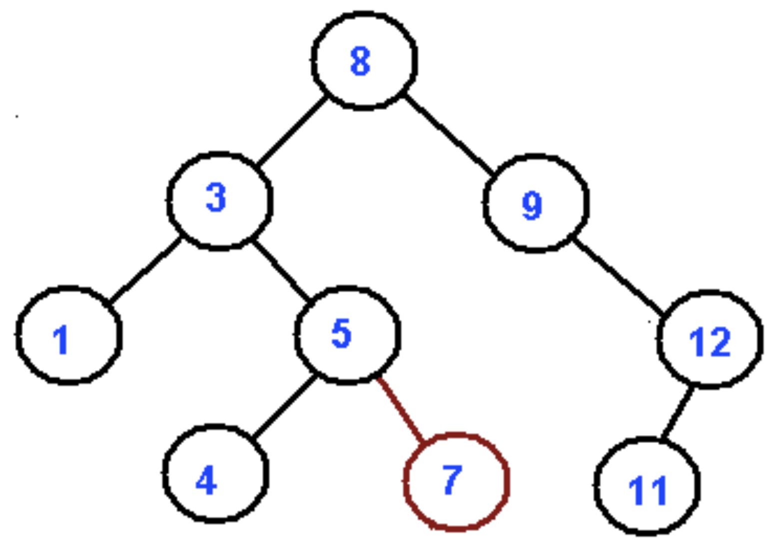
```
public class LinkedBinaryTree<E
extends Comparable<E>> implements
BinaryTreeInterface<E>
{
    /** The number of elements in the
tree */
    private int size;

    /** The root of the tree */
    private Node root;
```

Insertion

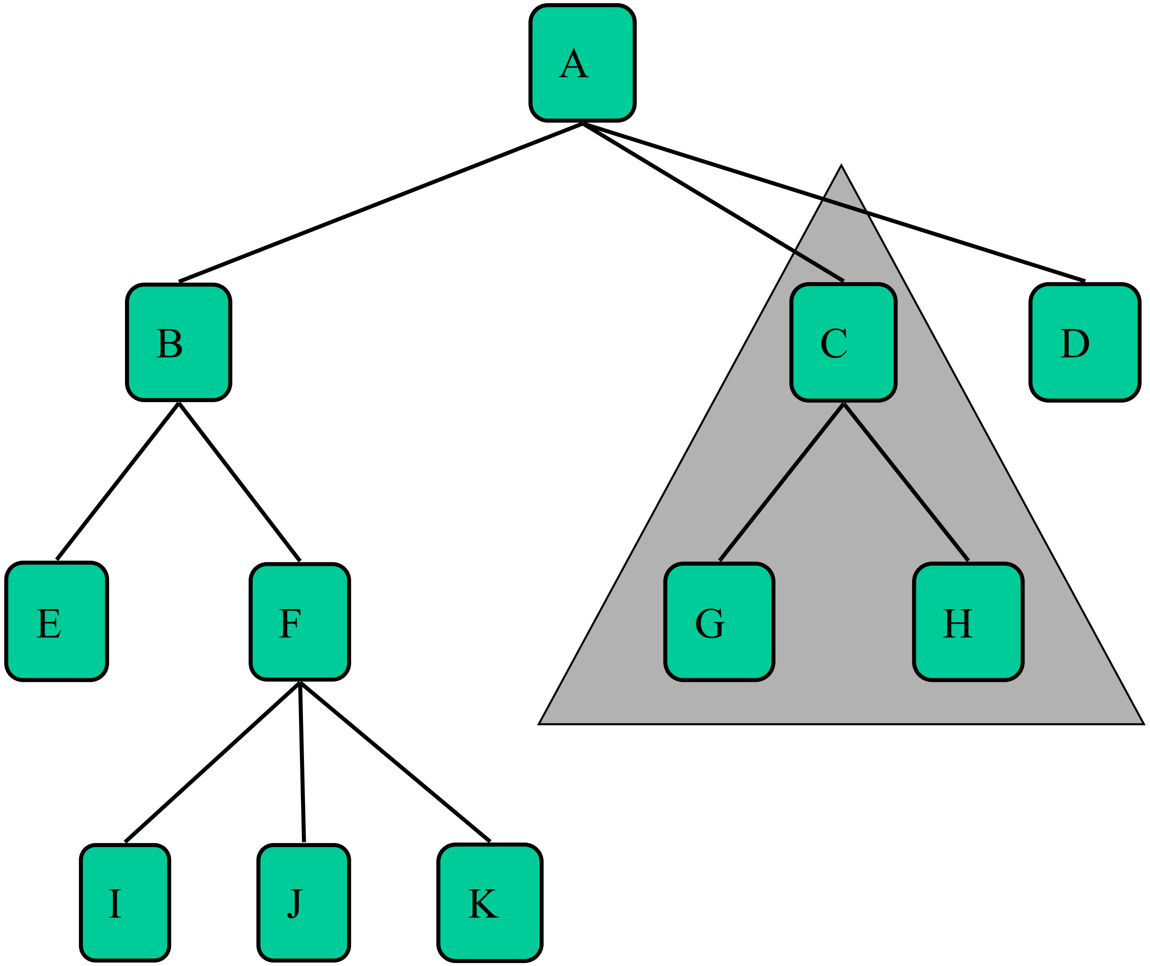
- smaller to the left, bigger to the right

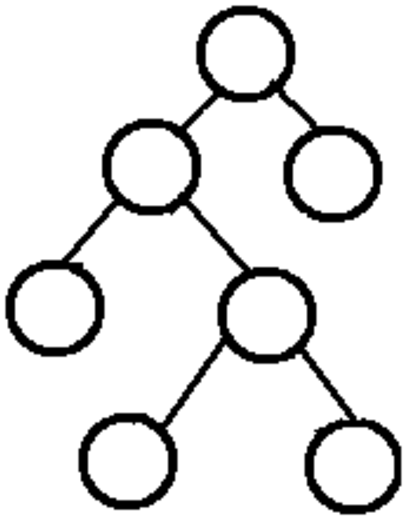


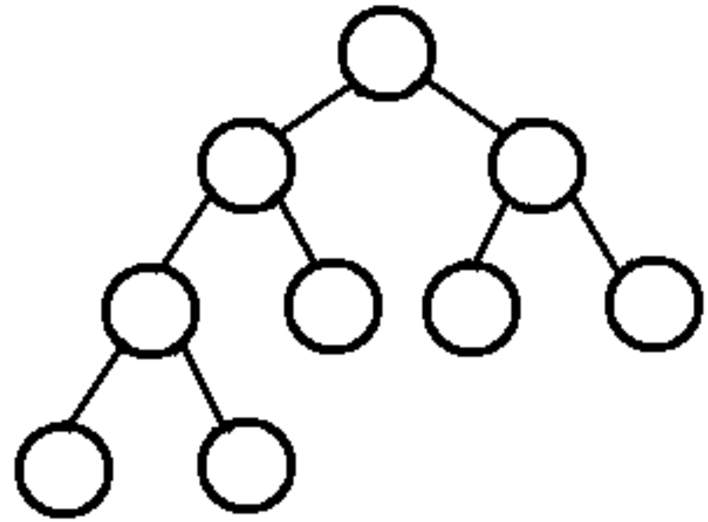


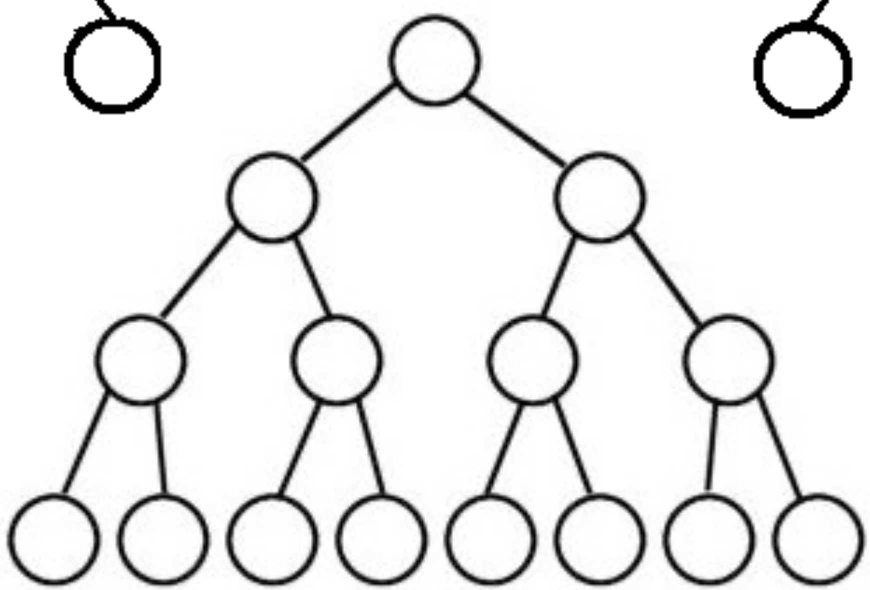
Draw some Binary Trees

- 11, 6, 8, 19, 4, 10, 5, 17, 43, 49, 31
- 6, 19, 10, 5, 43, 31, 11, 8, 4, 17, 49
- 4, 5, 6, 49, 43, 31, 19, 10, 11, 8, 17
- 17, 31, 8, 19, 43, 11, 5, 49, 10, 6, 4


A
B
C
D
E
F
G
H
I
J
K

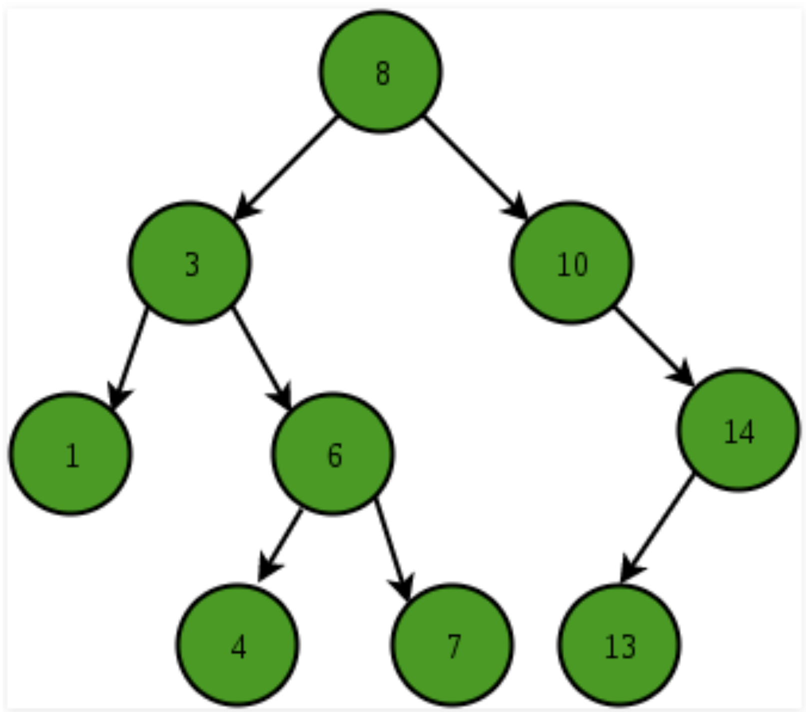






contains

- `boolean contains(E element);`
- returns true if found in the tree, false otherwise



Algorithm

- compare with root of **current subtree**
  - root is empty – return false
  - root == element – return true
  - root < element – recurse on right child
  - root > element - recurse on left child

Pseudo Code

```
findRec(root, key):
  if root == null:
    return false
  if root.key == key:
    return true
  if root.key > key:
    return findRec(root.left, key)
  else
    return findRec(root.right, key)
```

Recursive Helper Method

- The signature of `contains` doesn't allow any `Node` references (it cannot since `Node` is private)
- so define a private , recursive "helper" method.

```
public boolean contains(E element) {
   if (root==null) return false;
   return iContains(root, element)!=null;
}
private Node iContains(Node treepart, E toBeFound) {
     … }
```

write iContains at chalkboard

```java
private Node iContains(Node treepart, E toBeFound)
 {
 if (treepart==null) return null;
 int cmp = treepart.element.compareTo(toBeFound);
 if (cmp==0)
 {
     return treepart;
 }
 else if (cmp<0)
 {
     return iContains(treepart.left, toBeFound);
 }
 else // cmp>0
 {
     return iContains(treepart.right, toBeFound);
 }
 }
```

On chalkboard in class

insert

- `void insert(E element);`
- new node is always inserted as a leaf
- inserts to
  - left subtree if element is smaller than subtree root
  - right subtree if larger

Pseudo Code

```
insertRec(node, key):
  if node == null:
     add key to tree
  if root.key > key:
    node.left =
    insertRec(node.left, key)
  else
    node.right =
    insertRec(node.right, key)
```

Insert, with a helper

```
public void insert(E element)
{
   size++;
   if (root==null)
   {
       root=new Node(element);
       return;
   }
   iInsert(root, element);
}

private void iInsert(Node treepart, E toBeAdded) {
  … }
```

Write at chalkboard

```java
private void iInsert(Node treepart, E toBeAdded) {
    int cmp = treepart.element.compareTo(toBeAdded);
    if (cmp==0) {
        return; // the item is in the tree
    }
    else if (cmp<0) {
        if (treepart.left==null) {
        treepart.left=new Node(toBeAdded);
        }
        else {
        iInsert(treepart.left, toBeAdded);
        }
    }
    else // cmp>0 {
        if (treepart.right==null) {
        treepart.right=new Node(toBeAdded);
        }
        else {
        iInsert(treepart.right, toBeAdded);
        }
    }}
```

On
chalkboard
in class

Height / maxDepth

Again, using a recursive helper method

```
@Override
public int maxDepth()
{
   return iMaxDepth(root, 1);
}

int iMaxDepth(Node n, int depth) {
   …}
```