Quadtrees and Image Processing

CS 206 - Introduction to Data Structures

Assignment 9 - due Monday 5/13

This is a group project to be done in pairs. You should form a group of two as soon as possible.

1 Digital Images

A digital image is simply a rectangular grid of dots, often rendered as squares. Each dot is of a single solid color and is known as a pixel (short for picture element). The reason why we do not see individual pixels as squares is because they are very small. Resolution of a display device is measured by pixel density, or number of pixels per inch (PPI). Digital displays marketed from 2009 onwards have at least 100 PPI, with the newer and sharper ones reaching 200–300 PPI. Pixels are commonly arranged in a two-dimensional grid, the dimensions of which are specified by the image’s size in pixel resolution. For example, when we have a JPEG image of 1920x1080, it means the image has 1920 pixels in its width and 1080 pixels in its height. It has $1920 \times 1080 = 2,073,600$ total pixels and thus can also be referred to as a 2-megapixel image. Zooming into an image repeatedly will allow you to reach low enough resolution to see individual pixels, at which point you also lose most recognizable details in the image. This is called pixelation, also known as pixelization when the resolution lowering is done deliberately, a technique often employed in censorship.

![Figure 1: An image showing individual pixels rendered as squares](image)

In Computer Graphics, an image is also known as a bitmap, and is a data structure as well as an image representation corresponding to a spatially mapped array of bits, a natural representation of a pixel grid. The color representation of a pixel typically requires more than a single bit (which can only represent two colors, black and white), and the number of bits used per pixel is known as color depth. Colors are encoded as integers from 0 to 255, where 0 is black (no color) and 255 is full color and use the RGB color model in which red, green and blue are (added) blended together to produce a broad spectrum of colors. Consult the wikipedia page [https://en.wikipedia.org/wiki/RGB_color_model](https://en.wikipedia.org/wiki/RGB_color_model) for more details on RGB colors. Figure 2 shows three pixels with their
individual RGB color values and indices at which they are stored in the pixel grid. For example, the leftmost pixel can be found at row 81, column 123 and is colored with a red value of 137, green value of 196, and blue value of 138. Note that an RGB value of (137, 137, and 138) would be light gray, and thus the combination of (137, 196, 138) comes out light green.

Figure 2: Three pixels

2 Image Processing

Now that you see an image as an array of colors (given as integer triples), image processing is as simple as looping over this array and changing the numbers! This is known as “filtering”. Some simple image filters are for example:

1. negative - change each pixel (r, g, b) to (255-r, 255-g, 255-b)
2. grayscale - change each pixel (r, g, b) to (c, c, c) where c = r*0.3+g*0.59+b*0.11
3. tint - given a tint color (R, G, B), scale each pixel color (r, g, b) to (r/255*R, g/255*G, b/255*B). Note that full white (255, 255, 255) will become exactly (R, G, B), and everything else will be scaled proportionally between 0 and R/G/B.

2.1 Convolution Filters

A another popular technique is to compute a pixel’s color based on its immediate neighbors, including itself. We will base our discussions of these filters on a 3x3 neighborhood, the smallest and simplest; however, in practice neighborhoods can be larger, as well as differently shaped than a square (box). A 3x3 box filter computes the pixel color via a weighted average of all 9 pixels using some predetermined weights. If we consider an input pixel $\text{input}[i, j]$, then the weighted average can be defined as:

$$
\text{output}[i, j] = w_1 \times \text{input}[i - 1, j - 1] + w_2 \times \text{input}[i - 1, j] + w_3 \times \text{input}[i - 1, j + 1] + w_4 \times \text{input}[i, j - 1] + w_5 \times \text{input}[i, j] + w_6 \times \text{input}[i, j + 1] + w_7 \times \text{input}[i + 1, j - 1] + w_8 \times \text{input}[i + 1, j] + w_9 \times \text{input}[i + 1, j + 1]
$$

The choice of the weights (kernel) has dramatic effect on the resulting image. One popular and useful kernel is edge detection, with the weights set as:
That is:

\[
\text{output}[i,j] = -\text{input}[i-1,j-1] - \text{input}[i,j-1] - \text{input}[i+1,j-1] - \text{input}[i-1,j] + 8 \times \text{input}[i,j] - \text{input}[i+1,j] - \text{input}[i-1,j+1] - \text{input}[i,j+1] - \text{input}[i+1,j+1]
\]

Edge detection works by enhancing the differences between the center pixel and those that surround it. When you perceive an edge in an image, you are simply noticing that there is a (sharper) change in color/brightness. Thus, edge detection works by setting a pixel to black (0) if it’s not very different from its neighbors, and trend towards white (255) the more different it is. Consider what happens when the filter above is applied to an area of similar colors. The sum will trend towards zero. If the pixel is high contrast and bright, the sum will become more positive.

Figure 3: edge detection

3 Quadtree

A quadtree is a tree data structure in which each internal node has exactly four children and is most often used to partition a two-dimensional space by recursively subdividing it into four quadrants. On an image, a quadtree recursively divides the image into four subimages and stops when some criteria are met or reaching a single pixel. Each node
either stores the color of the pixel (if it’s a single pixel), or the average of the colors of all pixels in the quadrant.

3.1 Full Quadtree

When storing a full image as a quadtree, one could keep subdividing until all pixels at a node have the same color. Instead, for this project, we’ll choose a specific height for the resulting quadtree (we can think about this as being similar to the resolution of the image). This should be given as input, so a full quadtree with a height of 2 will have one root node, four children, and 16 grandchildren for a total of 21 nodes.

3.1.1 Rotation Filter

More to come soon.

3.2 Quadtree Image Compression

Instead of storing a quadtree with a specific height, we could store a compressed version of the image using a quadtree. A quadtree decomposition can be used to compress an image in stages as shown in Figure 5. This computes a hierarchy of images that represent a high quality image with increasingly more details (in the right places). The idea is to split only in those quadrants where the colors of the children differ greatly (according to some threshold) from that of the parent. Note the quadtree is selective and regions we choose to not split on presumably already have more or less the same colors and can afford to be blurred into the mean color. When the subdivision stops, the compressed image can be recovered from the leaves of the quadtree. Different settings of the difference threshold will generate compressions at different resolutions.

![Figure 5: Animated quadtree compression of an image step by step](image)

Besides compression, there are many applications for image hierarchies - a form of level-of-detail techniques in Computer Graphics. In computer games for example, if a player is far away, only low-res textures are applied on objects and as the player gets closer, textures of higher and higher resolutions are swapped in.

The algorithm is: split the image into four subimages, for each current node \( n_i \), calculate the mean color \( C_i \) and mean squared error \( E_i = \frac{1}{N^2} \sum_{x=1}^{N} \sum_{y=1}^{N} |n_i(x, y) - C_i|^2 \).

The error \( E_i \) is the average of the cumulative squared error between the compressed representation \( C_i \) and the original, which is computed as the squared Euclidean distance.
between the mean color \( C_i \) and all original pixel colors in the quadrant \( n_i \) of size \((N \times N)\). That is, for a pixel with color \((r, g, b)\), the squared error should be computed as \(((r - C_i.r)^2 + (g - C_i.g)^2 + (b - C_i.b)^2\). The usual square root is not needed. If a subimage has error greater than some threshold, split into four further subimages.

Figure 6: Quadtree image compression, different thresholds

4 Requirements

1. Given an input image, create a quadtree decomposition of the image and compute 8 compressed images of increasing resolutions as explained in 3.2. These 8 images should be generated at compression levels 0.002, 0.004, 0.01, 0.033, 0.077, 0.2, 0.5, 0.75. The compression level is defined as the number of leaves in the quadtree divided by the number of the original pixel count. Note lower levels represent higher compression ratio. The computed fractions do not have to equal the numbers exactly, approximately is fine.

2. Edge detection is expensive, since the filters require multiple (9 for 3x3, 25 for 5x5, etc) operations per pixel. With high resolution digital images having pixel counts in the 10s of millions and results needed in real time, it is often preferred to only apply the filter in important areas, which is where the quadtree comes in. Based on the same quadtree decomposition of an image, apply the edge detection filtering as explained in 2.1 only to those nodes of a sufficiently small size and replace the color of the larger nodes with black.

3. Rotation filter (more information to come soon)

4. Design your own filter based on the quadtree decomposition and do something interesting to the image. Explain your design in your README.

5. For debugging purposes, implement the ability to show the outline of the quadtree cells as shown on the left half in Figure 6. This is a required functionality requested by the commandline flag \(-t\). You should implement this BEFORE your program is
fully functional, as it is designed to help you debug. Programming it just to satisfy the requirement is the wrong order!

6. It is acceptable to only work on square images. If the input image is not square, output appropriate error message and exit.

5 Images Formats

In this assignment, our input will be images given in the Portable Pixel Map (PPM) format, which is a simple text file listing colors of each pixel in an image, as explained here: [https://en.wikipedia.org/wiki/Netpbm_format](https://en.wikipedia.org/wiki/Netpbm_format) Your output will be image(s) in the same format. Note that there are two versions of PPM, P3 (also known as plain or ascii) and P6 (binary). P6 is much more widely found because it is more space efficient and less prone to parsing difficulties. On the other hand P3 is plain text and thus easier to debug. We will be using the P3 format. Note that P3 doesn’t specify the number of pixel colors per line, thus you should be lenient when parsing P3 format and accept anything that looks remotely like a P3. Most image readers will convert to/from ppm to another other standard image format. On Linux, the command is called `convert`, which is part of the imagemagick software suite. An image called `test.png` can be converted using the following command: `convert test.png -compress none test.ppm`. You can substitute `png` for any other popular image format extensions such as `jpg` and `tif`. For more information, consult the manual pages using `man convert`.

A collection of test images can be found at `~dxu/handouts/cs206/data/a9/ppms`.

6 Command Line Input

You will receive an image file on the command line following the `-i` flag as your input: `Java Main -i test.ppm`. In addition, support the following flags:

1. `-o <filename>` indicates the root name of the output file that your program should write to
2. `-c` indicates that you should perform image compression
3. `-e` for edge detection
4. `-r h` for the rotation filter, where `h` is the requested quadtree height
5. `-x` for running your own filter
6. `-t` indicates that output images should have the quadtree outlined

For example:
`java Main -c -i test.ppm -o out` will generate 8 compressed images of `test.ppm` named `out-1.ppm`, `out-2.ppm`, ..., `out-8.ppm`, where `out-1.ppm` is the image with the lowest resolution/highest compression and so on. `java Main -e -i test.ppm -o out` will generate one output image called `out.ppm` which is result of applying edge detection to `test.ppm`. You may assume that only one of `-c`, `-e`, `-r` or `-x` will be given. However, `-t` may or may not be present on any filter.
7 Where to Start

At this point, you should be able to design a class structure on your own, so we do not give you specific suggestions here. However, here are some hints about tasks that you should make sure you can perform:

1. Be able to read in an image from a given file name and write out an image to a given file name.
2. Take an image and generate a quadtree (both compressed and not compressed).
3. Apply filters to an image.

8 Electronic Submissions

1. **README**: The usual plain text file README

   Your name:
   
   **How to compile**: Leave empty if it’s just javac Main.java
   
   **How to run it**: Leave empty if it’s just java Main
   
   **Known Bugs and Limitations**: List any known bugs, deficiencies, or limitations with respect to the project specifications. Documented bugs will receive less deduction versus uncaught ones.
   
   **Discussion**: Design of your own filter

2. **Source files**: all .java files

   **DO NOT INCLUDE**: Please delete all executable bytecode (.class) files prior to submission.

   To submit, store everything (README and source files) in a directory called A9. Then follow the directions here:
   https://systems.cs.brynmawr.edu/Submit_assignments